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The Base Surge : The Mechanism of Fall - Out

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The Base Surge : The Mechanism of Fall-Out

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Summary

As a result of the underwater nuclear explosion at Bikini (test Baker), a heavy mist, called the Base Surge, spread rapidly from the base of an opaque vertical column of fine water drops thrown into the air around the point of the explosion. This mist eventually covered an area of seven square miles. The area covered by a base surge is controlled by two factors, first the initial rate of spread caused by the weight of the mist, and secondly the checking of the spread as the density of the mist decreases when its water content is precipitated to the ground as "rain". The first factor has been studied both theoretically and by means of model experiments, and has been discussed elsewhere; the present report is a discussion of the mechanism of fall-out of the water content of the surge.

The results of rather crude theoretical studies of the problem indicate that the fine water droplets initially present in the mist at Bikini should have coagulated to large raindrops by a time which agrees well with that at which rain was observed to fall from the bottom of the spreading surge. The early stages of droplet growth appear to be controlled mainly by a process of evaporation and condensation of water vapour, but most of the growth occurs subsequently and is caused by collisions between droplets of differing sizes which fall under gravity at differing velocities relative to the surrounding air.

Introduction

When an atomic bomb is exploded at shallow depth under water, the force of the explosion creates a large volume of mist in the air immediately above and surrounding the point of the explosion. Since the average density of the mist is appreciably greater than that of the surrounding air, it collapses back on to the sea or ground, spreads rapidly, and may ultimately cover an area of several square miles. The name given to this phenomenon is the "Base Surge" because, on the first occasion when it was observed experimentally, the mist spread outwards from the base of a vertical column of air, water vapour and water droplets thrown upwards by the force of the explosion.

Many of the fission products from a nuclear explosion will be dissolved in the water droplets of the mist, and hence the area covered by the base surge becomes severely contaminated; the importance of understanding all aspects of the phenomenon will therefore readily be appreciated. Estimates of various kinds must be made, and there is little hope of making actual full-scale experiments with atomic bombs to check their accuracy. There is, however, one experiment in which fairly comprehensive data were obtained, namely, the second explosion (test Baker) at Bikini.

During the early stages of the spreading of the base surge, possibly up to times 2 or 3 minutes after the explosion in the case of test Baker, one can treat the motion as if the mist were an incompressible fluid of density greater than that of the surrounding air. Some mathematical investigations and a number of model experiments have been made in this Establishment, in order to determine the hydrodynamics of this part of the motion, and we now understand it in considerable detail. In particular, analysis of the model experimental results has shown that the main features of the observed motion of the Bikini base surge could be accounted for if the total liquid water content of the mist had been on the average about $3 - 4 \times 10^{-4}$ gm./cm.^{3}, so that the density of the mist was about 30% greater than that of air.

At Bikini, rain was observed to fall from the bottom of the spreading surge at about 3 minutes from the time of the explosion and, almost simultaneously, the surge rapidly slowed down and finally stopped spreading after about a further 2 minutes. Were it not for this precipitation of the water content of the mist, the Bikini base surge would undoubtedly have spread over a much greater area. Indeed, in perfectly still atmospheric conditions, it is difficult to conceive of any mechanism which would stop the spreading of a column of fluid, of density greater than that of air, released from rest.

The area contaminated by the base surge is therefore controlled by two factors. The first is the initial rate of spread caused by the weight of the mist. The second is the checking of the spread as the density of the mist decreases when fall-out occurs, for, as soon as the density of the mist has decreased to approximately that of the surrounding air, the driving force will have disappeared and the motion will be rapidly damped by frictional forces. The first factor may be regarded as understood; the second is not. The present paper contains a tentative and greatly over-simplified account of a possible mechanism of fall-out. The numerical results of a theoretical analysis of the problem are summarized and discussed in Section 1; the succeeding Sections 2, 3 and 4 contain more detailed treatments of various aspects of the problem; Section 5 inverts some of the arguments and makes some inferences about the initial droplet size distribution.

1. General Discussion of the Mechanism of Fall-Out

An explosive charge detonating just below the surface of water makes a bubble with a thin skin at the top. As the skin blows out and breaks, the gaseous products of the explosion vent through the water and break it into drops. These drops are carried upwards by their momentum against the resistance of the air, and, in the process, the drops are broken into finer droplets. The size of the droplet cloud produced in this way is not large; very approximately, the cloud is hemispherical of radius 20 - 30 times the charge radius. Meanwhile, the bubble or "crater" in the water increases steadily in size, and the water near the edge of the bubble, near the surface, shoots upwards. At this stage, the pressure inside the cavity is less than atmospheric, and the effect of this partial vacuum is to cause coalescence of the curtain or sheet of water rising from the edge of the crater. Thus, a jet, or plume, is formed. The plume caused by a small explosion (ounces to tons of ordinary H.E.) passes through the cloud of fine droplets coming from the disintegration of the water near and above the charge, and rises to a great height. Photographs of shallow underwater explosions taken on a very still day show the mist near the base, and the thin plume rising to remarkable heights through the mist. Once again, the water in the plume, or jet, breaks into drops, but for ordinary amounts of H.E. charge, these drops are only of fine or very fine raindrop size. They are much coarser than the fine basal mist.

When the explosive charge is very large, the situation changes to some extent. The basal mist is still formed and its dimensions are scaled up from those of the smaller charges. The jet, or plume, is again formed, and the cross-section of the jet after formation is accurately scaled up from smaller charges. The velocity of the jet is identical with that from smaller charges. But for the effect of gravity, the height reached by the plume would also scale, but simple arguments show that gravity does in fact greatly reduce the height, and in the case of the Bikini underwater bomb, reduced the height from the scaled value of 30,000 ft. to 5,000 ft. Now the basal mist was at least 1,800 ft. across and 1,000 ft. high, so that the plume was not much higher than the basal mist.

Neglecting for the moment the effect of gravity, so that the height of the plume scales, we see that unit volume of the plume, rising with the plume, must sweep through a volume of air which is proportional to the linear dimensions of the charge. The degree of fineness to which the water in the plume is reduced must increase with the amount of air passing through unit cross-section. Thus, the water coming from the plume caused by an underwater explosion of an atomic bomb may be expected to be reduced quickly to a mist of fine particles, and the particle radius might be as small as that in the basal mist.

The velocity of the plume at Bikini when it reached 300 metres height was of order 1000 metres per second. The radius of a water droplet which is small enough to survive an air current of 1000 metres per second is about 2 x 10^{-5} cm. Thus we seem to be on very firm ground if we assume that the water droplets caused by the Bikini explosion could not have been smaller than 10^{-5} cm. in radius. We shall therefore proceed on the assumption that 10^{-5} cm. is a lower limit, recognising that the actual figure may have been considerably larger.

Lane, Prewett and Edwards (Porton Technical Paper 115) following up some experiments made by Penney, have measured the mean particle radius of droplets of various liquids shattered by intense air currents produced by a blast gun. They were not able, with their apparatus, to make droplets smaller than 6 x 10^{-4} cm. even with air currents several hundred metres per second in speed. However, their air currents lasted for such short times that their evidence is not really applicable to our case. The point is recognised that some of the water thrown up at Bikini may have been no finer than 10 microns radius. Our object is to show that even if the initial mist were extremely fine, various physical processes soon increase the particle size, and, within a few minutes, most of the water falls out as "rain".

It was observed at Bikini that heavy rain, in the form of typical large raindrops of radius 0.1 - 0.25 cm., began to fall from the bottom of the base surge at about 3 minutes after the explosion, and these raindrops must have been formed from the fine droplets initially present in the mist by a process or processes of droplet growth. The remainder of the present report is a quantitative discussion of a possible physical mechanism of droplet growth under conditions similar to those which are believed to have occurred in the Bikini base surge. The phenomenon is presumably at least partly analogous to more common examples of the development of rain within mists, or even clouds, under appropriate meteorological conditions. The theories of droplet growth discussed below have therefore been developed from earlier theoretical studies by Smoluchowski and Langmuir, who were of course interested primarily in other applications of their theories. The present theoretical treatment is thus new only in points of detail, but the numerical results obtained differ widely in many respects from those deduced for analogous problems, on account of the extreme meteorological conditions in the base surge; for example, the relatively short time needed for the formation of raindrops within the surge appears to be due essentially to the relatively high concentration of liquid water.

The rate of growth of droplets which are of order 10^{-5} cm. radius appears to be due largely to two distinct physical processes, namely,

(i) coagulation due to the Brownian motion of the droplets, and

(ii) a statistical process of evaporation and condensation of water vapour.

The Brownian motion of fine droplets, which leads to coalescence between colliding droplets, has been studied intensively, both theoretically and experimentally; the theoretical predictions are adequately supported by experimental measurements, and the physical basis of this mechanism of droplet growth may therefore be assumed well understood. The alternative process is caused essentially by surface tension, which increases the vapour pressure in the droplets above that of saturated vapour. Hence droplets tend to evaporate even when surrounded by a saturated medium, and the smaller the droplet the greater is this tendency to evaporate; the surrounding medium then becomes super-saturated, vapour will re-condense on to the larger droplets, and the average droplet size therefore increases steadily with time.

The theory of coagulation by Brownian motion predicts that the rate of growth of very fine droplets is exceedingly rapid, and that the distribution of droplet sizes quickly becomes almost independent of the initial form of the distribution function. Summarizing, the radius of the average-sized droplet at time t after the formation of the mist is given approximately by

$$\overline{a} \sim 0.6 \left[WKt \right]^{1/3}, \qquad (1.1)$$

where W is the total concentration of liquid water ($\sim 3.5 \times 10^{-4}$ gm./cm.^{3} in test Baker), and K is the coagulation coefficient, which is normally of order 10^{-9} cm.^{3}/sec.; for example, with these numerical values of W and K, $\overline{a} \sim 1.6$ microns at 60 secs. after the formation of the mist. In addition, the theory predicts that the total number of droplets of mass greater than $x\,\overline{m}$, where $\overline{m}$ is the mass of a droplet of radius $\overline{a}$, at time t is given by

$$N_x = N e^{-x}, \qquad (1.2)$$

where N is the total number of droplets per c.c. in the mist at time t, and that the total mass concentration of these droplets is

$$W_x = W (x + 1) e^{-x}. \qquad (1.3)$$

Thus, for example, the number of droplets of radius greater than $2\bar{a}$ is $N e^{-8} \sim 3 \times 10^{-4} N$, and these droplets contain a fraction $9 e^{-8} \sim 0.3\%$ of the total liquid water in the mist.

The theory of droplet growth by the evaporation and condensation of water vapour also predicts that the rate of growth of very fine droplets is exceedingly rapid, and leads to the result

$$\bar{a} \sim 10^{-4} t^{1/3}. \qquad (1.4)$$

The coefficient 10^{-4} is obtained from the appropriate numerical values of such quantities as the surface tension of the droplet and the vapour pressure of saturated water vapour, and is therefore not an absolute constant, but can probably be taken as constant over the range of atmospheric conditions likely to occur with most atomic bomb explosions; in particular, to the order of accuracy of the theory, $\bar{a}$ is independent of the water concentration of the mist so long as the surrounding medium is saturated. The average droplet radius is therefore of order 4 microns at 60 secs. after the formation of the mist. There seems to be little point in developing this theory sufficiently far to enable the distribution of droplet sizes within the mist to be determined. However, in view of the similarity in form between equations (1.1) and (1.4), and on account of the inherent stability of both processes of growth, it would appear at least plausible to assume that this distribution is the same as that stated above for coagulation due to the Brownian motion of the droplets.

Both processes of growth are of course operative simultaneously, and the above numerical results indicate, although not entirely conclusively, that evaporation and condensation of vapour is a more important cause of droplet growth than is Brownian motion. The combined effect of the two processes is therefore probably not significantly greater than the effect of the latter alone, even if the effect of Brownian motion has been under-estimated. For example, the rate of coagulation due to Brownian motion would certainly be increased by turbulence in the base surge, which would increase the coagulation coefficient, but it appears difficult to make any reasonable quantitative estimate of the magnitude of this effect. In addition it seemed possible at one time that the Brownian velocity of the droplets would have been increased significantly by the recoil momentum from the gamma-rays emitted by radio-active fission products dissolved in the droplets, but rough calculations have shown that this effect is quite negligible.

The main conclusion to be drawn from this part of the work is that Brownian motion, and evaporation and condensation of water vapour, will both quite rapidly produce an appreciable number of droplets of a few microns radius within a heavy mist, but that neither mechanism can account for the formation of large raindrops, of say 1000 - 2500 microns radius, within a period of a few minutes, or even hours. In order, therefore, to account for the heavy rain which was observed at Bikini, it would appear certain that other processes much more fruitful in causing the growth of relatively large droplets must be invoked. Two possible mechanisms suggest themselves, namely,

(iii) the differential falling velocity of droplets of differing sizes, leading to overtaking and capture, and

(iv) electro-static forces caused by the intense ionization continuously produced by gamma-radiation from the fission products.

Experimental evidence suggests that charged and uncharged smoke particles in general coalesce at much the same rate, and that the electrified smokes, however formed, usually contain equal numbers of positively- and negatively-charged particles. As, however, the gamma-radiation emitted from the fission products dissolved in a droplet will be scattered mainly by electrons, it seems likely that the droplets will all be charged positively, on account of the ejection of electrons, whilst the surrounding air and water vapour molecules will be charged negatively. If this view is correct, the electro-static forces will impede coagulation. For the time being, however, it is proposed to neglect electro-static forces, and treat the problem of coagulation as if the droplets were uncharged, although it is recognized that errors may be introduced by this assumption.

A suspension of droplets whose density is greater than that of the surrounding medium will fall under gravity relative to the medium, and the velocity of fall increases with the size of the droplets. Thus larger droplets will tend to overtake and capture smaller droplets, and hence steadily increase in size; this effect will be called coagulation due to differential falling velocities. An elementary and much over-simplified theoretical study of this mechanism of coagulation, applied to the case of a suspension of water droplets in air, leads to the following results. If a single droplet, initially of radius r_0, is falling relative to a uniform mist of smaller droplets, each of radius a, the radius r of the larger droplet at a subsequent instant is related to the time t and distance s of fall by the equations

$$s = 4(r - r_0)/W\,\overline{E}, \qquad (1.5)$$

and

$$0.3 \times 10^6\, W\overline{E}t = 1500\left[\sqrt{r} - \sqrt{r_0}\,\right] + k/r_0$$

$$- (1/2a)\ \log_e \left[(r + a)/(r - a)\right]. \qquad (1.6)$$

In these equations, W is the mass concentration of liquid water in the form of the smaller droplets, $\overline{E}$ is the average "efficiency of collection" during the process of growth, and k is an empirical constant which is probably of order 1.1 - 1.2. Except during the early stages of growth, the final term on the right of equation (1.6) can be replaced by $(1/r)$.

The efficiency of collection E is the fraction of the smaller droplets, within the volume swept out by the larger droplet, which coalesce with the larger droplet, and is less than unity since the air flow round the larger droplet deflects some of the smaller droplets from its path. The main difficulty in the application of this theory is in fact the determination of the appropriate value of the average efficiency of collection $\overline{E}$. Theoretical calculations indicate that E remains zero until the larger and smaller droplets have both become relatively large, and that no appreciable amount of coagulation due to differential falling velocities could occur until the average droplet is of order 10 microns radius; on the basis of the previous discussion, it would appear unlikely that such large droplets could be produced in significant quantities by alternative mechanisms of coagulation within a time of only a few minutes. On the other hand, these theoretical calculations are quite likely to be in error, and to under-estimate the efficiency of collection, if the larger and smaller droplets are of comparable size, and a few model experiments have indicated, very roughly, a value $\overline{E} \sim 0.2$. In lieu of better information, this value has been adopted in the present calculations. It is unlikely to be an under-estimate during the critical early stages of coagulation, and may be a gross over-estimate; if this is the case, the actual time needed for droplets to grow to large raindrops will be very much longer than the estimates quoted below.

explosion, and that the base surge finally came to rest after about a further 2 minutes.

The suggested mechanism of fall-out is therefore that of coagulation of fine droplets to large raindrops caused by two or three quite distinct mechanisms of droplet growth. The rate of growth during the early stages of coagulation is due mainly to two physically distinct but mathematically analogous processes of droplet growth, namely, Brownian motion and the evaporation and condensation of water vapour, the latter of which was probably dominant under the conditions which are estimated to have held at Bikini. The later stages of growth are caused almost entirely by the process of coagulation due to the differential falling velocities of droplets of differing sizes. The present mathematical analysis of the course of the growth of droplets is exceedingly crude, but it might be possible at a later stage to consider the problem in greater detail and derive the integro-differential equation satisfied by the distribution function of the size of the droplets, as a function both of time and of height above some reference plane. The solution of this equation can probably only be obtained numerically, and hence this more exact approach to the problem will only be attempted when it is fairly certain that the calculation would be worth-while.

Finally, consider a hypothetical case of an underwater atomic bomb explosion in which a base surge is formed containing a concentration of liquid water z times the value in test Baker. We assume also that z is not too greatly different from unity; in fact, arguments which will not be presented here suggest that a base surge can only be formed at all if z lies within quite close limits, possibly only within the range 0.8 - 1.2. The time required for the initial stages of coagulation, mainly by evaporation and condensation of water vapour, is then, according to the previous arguments, almost independent of z, and may, for comparison, be taken as 60 secs. On the other hand, the time needed for coagulation due to differential falling velocities is inversely proportional to the concentration of liquid water, and may therefore be taken as $90/z$ secs., so the total coagulation time is $(60 + 90/z)$ secs. Further, the study of the hydrodynamics of the motion has shown that the area covered by the base surge up to a given time t is roughly proportional to zt, except during the early stages of the motion. Therefore, taking the area covered by the Bikini surge up to fall-out as unity, the corresponding area covered in the hypothetical case is $(0.6 + 0.4\ z)$. The area contaminated by the base surge should therefore increase, but rather slowly, with the liquid water content of the surge; even in the extreme case z = 2, the increase in area is only 40%, and, over the range $0.8 \leqslant z \leqslant 1.2$, the relative area only varies between 0.92 and 1.08.

2. The Coagulation of Fine Droplets by Brownian Motion

The rate of coagulation of fine particles suspended in a fluid has been studied theoretically by Smoluchowski (cf., for example, "Micromeritics", by J.M. Dallavalle, Pitman, 1948), who derived the following results for the rate of coagulation due to the Brownian motion of the particles. Suppose that the suspension initially contains N_0 particles per c.c., each of mass m_0 and equivalent radius a_0; then, after time t, the suspension will contain N_1 particles per c.c. of mass m_0, N_2 particles of mass $2m_0$, N_3 of mass $3m_0$, and so on, where

$$\left.\begin{aligned} N_1 &= N_0 / \left[1 + (t/t_c)\right]^2 \\ N_2 &= N_0 (t/t_c) / \left[1 + (t/t_c)\right]^3 \\ &\ldots\ldots\ldots \end{aligned}\right\} \qquad (2.1)$$

$$N_i = N_o(t/t_c)^{i-1} / \left[1 + (t/t_c)\right]^{i+1}.$$

The total number of particles per c.c. after time t is thus

$$N = \sum_{i=1}^{\infty} N_i = N_o / \left[1 + (t/t_c)\right], \qquad (2.2)$$

and hence the number of particles has been halved after time t_c.

Equation (2.2) can be written alternatively in the form

$$1/N - 1/N_o = t/N_o t_c = Kt, \qquad (2.3)$$

where K is called the coagulation constant, and Smoluchowski derived the further theoretical result

$$K = 4kT/3\mu, \qquad (2.4)$$

where k is Boltzmann's constant, T is the absolute temperature, and μ is the coefficient of viscosity of the fluid. This value of K is independent of both the size and concentration of the suspended particles, except in so far as these factors influence the viscosity μ. For particles suspended in dry air at atmospheric temperature, the theoretical value of K is of order 0.3×10^{-9} cm.^{3}/sec., and it has been found experimentally that K is usually about 0.5×10^{-9} for smoke particles in still air. Further, the viscosity μ, and hence K, is practically unaffected by the presence of water vapour in air under normal atmospheric conditions. On the other hand, it has been observed experimentally that K is increased markedly by atmospheric turbulence. It is, however, difficult to make reasonable quantitative estimates, which are applicable to the present problem, of either the scale of the turbulence or its effect on the coagulation constant, and it will therefore be assumed that K is thereby increased by a factor not exceeding 10.

It has been estimated, on the basis of theoretical and model experimental studies of the motion of the column of mist formed at test Baker, that the total concentration of liquid water initially present in the mist was about $3 - 4 \times 10^{-4}$ gm./cm.^{3}, and it has also been estimated that the force of the explosion would have broken this water into droplets of radius less than 10^{-5}cm.; the corresponding initial concentration of droplets would be $N_o \sim 10^{11}$ per c.c.

For times of order several seconds, the term $(1/N_o)$ in equation (2.3) can thus be neglected in comparison with Kt, and hence

$$N \sim 1/Kt, \qquad (2.5)$$

so the number of water droplets in the mist fairly quickly becomes independent of their initial size and number. For example, taking $K = 0.5 \times 10^{-9}$ and t = 60 secs., $N \sim 3 \times 10^7$ per c.c; if K is greater by a factor of 10 on account of atmospheric turbulence, the value of N would be reduced by the same factor. The mean mass of the droplets contained in the mist at time t is

$$\overline{m} = m_o N_o / N \sim WKt, \qquad (2.6)$$

where W $[= m_o N_o]$ is the total concentration of liquid water ($\sim 3.5 \times 10^{-4}$ gm./cm.^{3}) in the mist. The equivalent radius corresponding to this mean mass is

$$\overline{a} = \left[3WKt/4\pi\rho_W\right]^{1/3}, \qquad (2.7)$$

where ρ_W is the density (= 1 gm./cm.³ for liquid water) of each droplet. The value of $\overline{a}$ is of course also independent of the postulated initial droplet radius. Taking $K = 0.5 \times 10^{-9}$ and $t = 60$ secs., $\overline{a} \sim 1.4 \times 10^{-4}$ cm.; an increase of K by a factor of 10 would roughly double the value of $\overline{a}$.

When t is of order several seconds, $(t/t_c)\ [= N_0Kt]$ is large compared with unity, and hence, from equation (2.1),

$$\log_e (N_1/N_{j+1}) = j \log_e \left[1 + (t_c/t)\right] \sim j\,(t_c/t), \qquad (2.8)$$

so

$$j \sim (t/t_c) \log_e (N_1/N_{j+1}) = N_0Kt \log_e (N_1/N_{j+1}). \qquad (2.9)$$

The mass of each droplet of type j is thus

$$m_j = jm_c = WKt \log_e (N_1/N_{j+1}), \qquad (2.10)$$

which is equal to x times the mean mass $\overline{m}$ per droplet if

$$N_{j+1} = N_1 \ e^{-x}. \qquad (2.11)$$

The total concentration of droplets of mass greater than $x\overline{m}$ at time t is therefore

$$N_x = \sum_{i=j+1}^{\infty} N_i = N e^{-x}. \qquad (2.12)$$

Thus, for example, about 63% of the total number of droplets have mass not exceeding $\overline{m}$, about 23% have mass in the range $\overline{m}$ to $2\overline{m}$, about 8.5% have mass in the range $2\overline{m}$ to $3\overline{m}$, and so on. Further, the total mass of all the droplets each of mass greater than $x\overline{m}$ is

$$W_x = N\overline{m} \int_x^{\infty} ue^{-u}\,du = W(x+1)\,e^{-x}. \qquad (2.13)$$

Hence, in particular, about 26% of the total liquid content of the mist consists of droplets of individual mass less than $\overline{m}$, about 33% of droplets of mass in the range $\overline{m}$ to $2\overline{m}$, about 20% in the range $2\overline{m}$ to $3\overline{m}$, about 11% in the range $3\overline{m}$ to $4\overline{m}$, about 5% in the range $4\overline{m}$ to $5\overline{m}$, and so on. It should be noted that the relatively small number of relatively large droplets, of say m greater than $4\overline{m}$, contain a significant fraction of the total water content.

These results, which are well known, have been derived primarily to emphasize the fact that, starting from a suspension of droplets of uniform size, the process of coagulation quickly produces droplets whose mass varies in accordance with the distribution W_x, which does not vary with time. The same result would ultimately be obtained with any initial distribution of droplet sizes, for it is clear on physical grounds that the process of coagulation by Brownian motion is inherently stable.

The above analysis would appear to have shown fairly conclusively that, however small the initial size of the droplets in the mist formed at test Baker, coagulation due to Brownian motion would have produced fairly quickly an appreciable proportion of droplets of at least 1 micron radius, and that the radius of the average droplet was at least 1 - 2 microns at a time of about 1 minute after the explosion. Further, it is clear that Brownian motion alone could not produce an appreciable number of very much larger droplets during the few minutes before rain began to fall from the bottom of the base surge, for the radius of the average droplet is proportional to the cube root of the time; for example, after 5 minutes, the radius of the average droplet would probably not have exceeded 5 microns at most, and very few droplets would then have had radii greater than 10 microns.

3. The Growth of Fine Droplets by Evaporation and Condensation

An elementary theory of the rate of growth of the average droplet size in a suspension of liquid droplets, on account of evaporation and condensation, has been developed by I. Langmuir ("Supercooled water droplets in rising currents of cold saturated air", General Electric Company Report). The following discussion is based on Langmuir's theory, slightly modified at one point to enable the results to be applied to the present physical problem. We shall ignore the fact that the water was sea water and therefore contained salt. A more complete theory must take the salt into account.

The surface tension of a small liquid droplet exerts a considerable hydrostatic pressure which increases the vapour pressure in the droplet above that of saturated vapour. Hence the droplet will tend to evaporate even when surrounded by a saturated medium, and the smaller the droplet the greater will be this tendency to evaporate. The average life τ of a droplet of initial radius a in a saturated medium has been calculated by Langmuir, who derived the result

$$\tau = (1/6D\gamma e_W) \left[\rho_W RT/M \right]^2 a^3, \qquad (3.1)$$

where D is the diffusion coefficient, γ is the surface tension of the droplet, e_W is the vapour pressure of the saturated vapour, ρ_W is the density of the droplet, R is the gas constant, T is the absolute temperature, and M is the molecular weight of the vapour. For a water droplet in saturated air under normal atmospheric conditions, $D \sim 0.245$ cm.^{2}/sec., $\gamma \sim 72$ dynes/cm., $e_W \sim 17 \times 10^3$ dynes/cm.^{2}, $\rho_W = 1$ gm./cm^3, $R = 8.31 \times 10^7$ ergs/°C, $T \sim 288$°K and M = 18 gm. Therefore $\tau \sim 10^{12}\ a^3$, where τ is in secs., and a in cm. The average life of a droplet of 1 micron radius is thus only 1 sec.

Equation (3.1) is only valid when the radius r of the droplet is large compared with the mean free path of the surrounding air molecules, which is of order 10^{-5} cm. under normal atmospheric conditions. The corresponding theory for much smaller droplets has also been developed by Langmuir, and his results show, as would be expected, that the average life of such droplets is very much less than 1 sec. It is therefore unnecessary to consider this modified theory further in the present report, since the droplets to which it applies will be eliminated almost instantaneously, either by evaporation or, as was shown in Section 2, by coagulation due to Brownian motion.

In order to determine the rate of growth of the droplet size, Langmuir introduced the hypothesis that the fractional rate of decrease of the total number N of droplets per c.c. is inversely proportional to the life τ of the average sized droplet, that is, that

$$\frac{1}{N}\frac{dN}{dt} = \frac{-1}{\beta\tau} \sim \frac{-1}{10^{12}\ \beta\bar{a}^3}, \qquad (3.2)$$

where $\bar{a}$ is the equivalent radius of a droplet of mean mass $\bar{m}$, and β is an empirical constant. Experimental measurements of the rate of growth of droplets injected into air gave values of β approximately equal to unity, and this value will be adopted in the following discussion.

The product $N\bar{a}^3$ is proportional to the total liquid water content of the suspension, which in general will be a known or calculable function of the time, that is,

$$N\overline{a}^3 = f(t). \qquad (3.3)$$

Hence, differentiating logarithmically,

$$\frac{1}{N}\frac{dN}{dt} + \frac{3}{\overline{a}}\frac{d\overline{a}}{dt} = \frac{f'(t)}{f(t)}\ , \qquad (3.4)$$

and therefore

$$3\overline{a}^2\ \frac{d\overline{a}}{dt} - \overline{a}^3\ \frac{f'(t)}{f(t)} = 10^{-12}\ , \qquad (3.5)$$

which integrates to

$$\overline{a}^3 = 10^{-12} f(t)\left\{\int^t \frac{du}{f(u)} + \text{const.}\right\}. \qquad (3.6)$$

Langmuir applied this theory to determine the average droplet size in a rising current of air when the total liquid water content increased steadily with the time on account of a steady decrease in air temperature. In the present application, f(t) would certainly not have increased significantly with the time during the spreading of the base surge, and would in fact be decreased by turbulent mixing of the mist with surrounding drier air. It seems possible, however, that this effect will only be of importance fairly near the boundaries of the base surge during the few minutes before the water content of the surge precipitated in the form of rain, and that the total liquid water content of the core remained fairly constant during this period. It will therefore be assumed that f(t) is constant in the present application, and then equation (3.6) becomes

$$\overline{a}^3 = \overline{a}_0^3 + 10^{-12} t, \qquad (3.7)$$

where $\overline{a}_0$ is the initial radius of the average sized droplet. Further, since $\overline{a}_0$ is considered to be less than 10^{-4} cm., the term $\overline{a}_0^3$ is comparatively negligible after times of order several seconds, and hence, finally, $\overline{a} \sim 10^{-4}\ t^{1/3}$ cm., where t is in secs. In particular, the radius of the average-sized droplet is about 4 microns after 60 secs.

Unfortunately, this elementary theory of the growth of droplets by evaporation and condensation provides no direct information as regards the distribution of droplet sizes. However, since the physical mechanism of growth is inherently stable, it may reasonably be anticipated, by analogy with the discussion of Section 2, that the distribution function with respect to the mean will tend towards a form which is independent of the time. Further, noting that the average droplet radius is proportional to the cube root of the time both for coagulation due to Brownian motion and by evaporation and condensation, it would appear plausible to assume that the limiting distributions of droplet sizes are the same in each case.

Finally, since both mechanisms of droplet growth must of course be operative simultaneously, it is relevant to compare their relative importance in order to be able to estimate, at least crudely, their combined effect. Further, the most valid comparison would appear to be between the masses of the droplets produced in the two cases, since the droplet formed by coagulation has mass equal to the sum of the masses of the colliding particles. According to equation (2.6), the average droplet mass resulting from coagulation due to Brownian motion is of order $(0.2 - 2) \times 10^{-12} t$ gm., depending mainly on the assumed value of the coagulation constant. Similarly, from equation (3.7), the corresponding mass resulting from evaporation and condensation is of order $4 \times 10^{-12}t$ gm. These values are of course not sufficiently precise to enable firm conclusions to be drawn, but, tentatively, it would appear that evaporation and condensation is a rather more important mechanism of droplet growth than is Brownian motion. The combined effects of the two processes should therefore not be appreciably greater than the effect of evaporation and condensation alone, and hence, to the order of accuracy of the present estimates, the radius $\bar{a}$ of the average sized droplet, as a result of both processes operating simultaneously, is given by

$$\bar{a} = 10^{-4}\, t^{1/3} \text{ cm.} \qquad (3.8)$$

4. Coagulation of Droplets caused by Differential Falling Velocities

(a) The general equations. A suspension of droplets whose density is greater than that of the surrounding fluid medium will fall under gravity relative to the medium. Moreover, the velocity of fall will increase with the size of the droplets, and thus larger droplets will tend to overtake and capture smaller droplets, and hence steadily increase in size; this effect will be called coagulation due to differential falling velocities. This mechanism of the growth of droplets has been discussed in considerable detail by I. Langmuir ("Production of rain by chain reaction in cumulus clouds at temperatures above freezing", General Electric Company Report, dated 3.2.48), and the following discussion is based largely on Langmuir's formulation of the theory.

According to Stokes' law, the resisting force acting on a sphere of radius r moving at velocity V through a fluid medium is

$$R = 6\pi\mu r\, V, \qquad (4.1)$$

where μ is the coefficient of viscosity of the fluid. This formula is only applicable when the product rV is sufficiently small, and a more general result, applicable over the whole range of values of r and V, is

$$R = 6\pi\mu r\, V\,(C_D\, Re/24), \qquad (4.2)$$

where C_D is the drag coefficient of the sphere, a function of the Reynolds number Re only, which is defined as

$$Re = 2\rho_a\, r\, V/\mu, \qquad (4.3)$$

where ρ_a is the density of the fluid. The terminal velocity V_1 of a sphere falling freely under gravity is thus given by

$$(C_D\, Re/24)\; V_1 = \left[2(\rho_W - \rho_a)g/9\,\mu\right] r^2\,, \qquad (4.4)$$

where ρ_W is the density of the sphere, and g is the gravitational acceleration; in this equation, Re and C_D must of course be determined at the velocity V_1. Therefore, taking $(\rho_W - \rho_a) \sim 1$ gm./cm.^{3}, $\mu \sim 1.8 \times 10^{-4}$ gm./cm. sec., and $g \sim 981$ cm./sec.^{2}, which are appropriate values for the present problem,

$$(C_D\, Re/24)\; V_1 \sim 1.2 \times 10^6\, r^2 \text{ cm./sec.}, \qquad (4.5)$$

where r is in cm.

Now consider a single droplet of radius r falling at velocity V through a uniform mist of smaller droplets, each of radius a, of mass $m \left[= \frac{4}{3}\pi\rho_W a^3 \right]$, and of total concentration N per c.c. Except at positions fairly close to the larger droplet, where the air flow is disturbed by its motion, it may be assumed that the smaller droplets are all falling at their appropriate terminal velocities. Further, in the present application of the theory, the smaller droplets are at most a few microns in radius, so the air resistance to their motion is given by Stokes' formula, and their terminal velocity is $1.2 \times 10^6\, a^2$ cm./sec. The relative velocity of fall of the larger and smaller droplets is therefore

$$v = V - 1.2 \times 10^6\, a^2 , \qquad (4.6)$$

and the relative volume swept out by the larger droplet in unit time is $\pi r^2 v$. On the average, this volume will contain $\pi r^2 vN$ droplets of radius a, of which a proportion E will collide with, and adhere to, the larger droplet, where E is called the efficiency of collection. The differential equation for the radius of the larger droplet as a function of time is thus

$$\frac{d}{dt}\left[\frac{4}{3}\,\pi\rho_W r^3\right] = \pi r^2 vNEm,$$

which reduces, taking $\rho_W = 1$ gm./cm.^{3}, to

$$\frac{dr}{dt} = \frac{1}{4}\, vEW, \qquad (4.7)$$

where $W = Nm$ is the total concentration of liquid water in the mist in the form of droplets of radius a.

The equation of motion of the larger droplet is

$$\frac{d}{dt}\left[\frac{4}{3}\,\pi\rho_W r^3 v\right] = \frac{4}{3}\,\pi(\rho_W - \rho_a)r^3 g - 6\pi\mu r\, V(C_D\, Re/24),$$

which reduces to

$$rV(C_D\, Re/24) = 1.2 \times 10^6\, r^3 - 1.2 \times 10^3 \frac{d}{dt}\left[r^3 v\right]. \qquad (4.8)$$

Therefore, eliminating the time t between equations (4.7) and (4.8),

$$rV(C_D\, Re/24) = 1.2 \times 10^6\, r^3 - 300\, vEW \frac{d}{dr}\left[r^3 v\right]. \qquad (4.9)$$

Assuming for the moment that the efficiency of collection E is a function of r and v only, the integral of this last equation determines the velocity v as a function of the radius r of the larger droplet. The time of fall is then given by

$$t = \frac{4}{W}\int_{r_0}^{r} \frac{dr}{vE}\, , \qquad (4.10)$$

where r_0 is the initial radius of the larger droplet, and the distance fallen is

$$s = \int_0^t V\, dt = \frac{4}{W}\int_{r_0}^{r} \frac{dr}{E} + 1.2 \times 10^6\, a^2 t. \qquad (4.11)$$

The contribution from the final term of this equation is in general entirely negligible.

(b) The efficiency of collection. The efficiency of collection E by a sphere has been studied theoretically by Langmuir (loc. cit), who found that E is a function of a single non-dimensional parameter κ, defined by

$$\kappa = 2\rho_W a^2 v/9\mu r \sim 1.2 \times 10^3 a^2 v/r. \qquad (4.12)$$

In addition, Langmuir derived the result that E is zero unless κ exceeds a critical value, which is of order unity when the flow round the larger sphere is of Stokesian type. Now the relative velocity v cannot exceed the terminal velocity of the larger droplet as determined by Stokes' formula, that is, $1.2 \times 10^6 r^2$ cm./sec., and hence the parameter κ is less than $1.5 \times 10^9 a^2 r$. The result shows that the efficiency of collection, as calculated by Langmuir, is zero unless both droplets are of relatively large size; for example, if a = 3 microns, E is zero unless r exceeds about 70 microns, whilst, if a = 5 microns, r must exceed about 25 microns.

It has already been estimated in Section 3 that the radius of the average-sized water droplet in the Bikini base surge was of order 4 microns at one minute after the explosion. According to Langmuir's results, coagulation due to differential falling velocities must then have been still a relatively rare occurrence, for it could only have taken place between droplets both of which were appreciably larger than the average size. In fact, it would appear that very little coagulation of this type could have occurred until the average droplet radius had attained a value of about 10 microns; if the dominant alternative mechanism of coagulation was that of evaporation and condensation, no appreciable amount of coagulation due to differential falling velocities could have occurred until after at least 10 minutes from the time of the explosion. On the other hand, it was observed experimentally at Bikini that heavy rain began to fall from the bottom of the base surge at about 3 minutes after the explosion.

Langmuir's calculation of the efficiency of collection by a sphere is based on the determination of the trajectory of the smaller droplet in the field of the airflow round the larger droplet, and contains two fundamental assumptions, first, that the field of flow is unaffected by the presence or motion of the smaller droplet, and, secondly, that coalescence always occurs if the two droplets collide. Any disturbance of the flow due to the smaller droplet will tend to increase the chance of collision, and hence increase the efficiency of collection, by an amount depending on and increasing with the ratio (a/r) of the two radii; this effect arises from the well known attractive force between two spheres moving along parallel trajectories through a fluid medium. Thus Langmuir's first assumption would appear to be valid when (a/r) is small, but may cause a serious under-estimate of the efficiency of collection when the two spheres are of comparable sizes. On the other hand, the efficiency of collection is clearly reduced if a proportion of the collisions does not lead to coalescence between the colliding droplets; it should, however, be noted that the aerodynamic forces will tend to prevent the separation of two droplets after a collision, and hence Langmuir's second assumption is probably rather more valid than is often thought to be the case.

The theoretical determination of the field of viscous flow round two spheres in relative motion is a very complicated problem, which has only been solved in a few special cases, in particular, when the centres of the spheres are moving along the same straight line (cf. "Hydrodynamik", by C.W.Oseen, Akademische Verlagsgesellschaft Leipzig, 1927). A detailed theoretical treatment of the efficiency of collection, when the two spheres are of comparable sizes, is therefore out of the question for the time being at least. Langmuir has, however, performed a few relevant experiments with glass spheres falling through very viscous liquids and found, very roughly, that the efficiency of collection was about 0.2, assuming of course that collisions always lead to coalescence.

As regards the present discussion, the only reasonable course which appears available for the time being is that E should be considered as an empirical constant, at least during the early stages of coagulation. In lieu of better information, the value E = 0.2 has been chosen; it seems unlikely that this value is too low during the critical early stages of coagulation, and it may be a gross over-estimate, but this point can only be settled satisfactorily by appropriate model experiments.

(c) <u>The velocity of fall</u>. Taking E = 0.2, equation (4.9) becomes

$$rV(C_D \, Re/24) = 1.2 \times 10^6 \, r^3 - 60 \, rW \frac{d}{dr}\left[r^3 v \right]. \quad (4.13)$$

Since the drag coefficient C_D is a rather complicated function of the Reynolds number Re, and is usually given in numerical form, this equation can only be solved exactly by numerical integration. It is however possible to derive two fairly simple analytic formulae which are approximately valid when the droplet is very small and very large respectively.

When the radius r is sufficiently small, it may be assumed first that the air resistance to its motion is given by Stokes' formula, and secondly that the second term on the right hand side of equation (4.13) is relatively negligible. The velocity of the droplet is then equal to the terminal velocity corresponding to its instantaneous size, that is, $V \sim 1.2 \times 10^6 \, r^2$ cm./sec. The first assumption is approximately valid when r does not exceed about 50 microns, and the second is also a good approximation over the same range, as may be shown as follows. From equation (4.13), an improved approximation to the droplet velocity is

$$V \sim 1.2 \times 10^6 \, r^2 \left[1 - 4 \times 10^8 \, W \, r^3 \right] .$$

Therefore, since W is of order 4×10^{-4} gm./cm.^{3}, the correcting term is of order $2 \times 10^5 \, r^3$, which is small compared with unity when r is less than about 50 microns.

When r is greater than about 1000 microns, the drag coefficient C_D is approximately constant and equal to about 0.4. Then, taking ρ_a = 0.0012 gm./cm.^{3}, $rV(C_D \, Re/24) \sim 0.2 \, r^2 V^2$. Further, when r is large the difference between the velocities v and V is quite negligible, and then equation (4.13) becomes

$$0.2 \, r^2 V^2 = 1.2 \times 10^6 \, r^3 - 60 \, VW \frac{d}{dr}\left[r^3 V \right], \quad (4.14)$$

which has the limiting solution

$$V^2 \sim 6 \times 10^6 r / \left[1 + 10^3 W \right] . \quad (4.15)$$

Taking $W = 4 \times 10^{-4}$, the droplet velocity is about 20% less than the corresponding terminal velocity, given by $V^2 \sim 6 \times 10^6 \, r$, and, to the order of accuracy aimed at in the present calculations, such a correction may be considered negligible. The droplet velocity is therefore approximately equal to the corresponding terminal velocity both when r is very small and when r is very large, and hence it would appear adequate to assume that V is equal to the terminal velocity of a droplet of radius r throughout the whole process of growth. This assumption is contained implicitly in Langmuir's discussion of the theory, and is in fact a much better approximation in his illustrative examples, since W is then very much smaller than was the case in test Baker.

The velocity of the larger falling droplet is therefore given by

$$V(C_D \, Re/24) = 1.2 \times 10^6 \, r^2, \quad (4.16)$$

and this equation has been solved numerically, over the whole relevant range of values of r, with the aid of a table, quoted by Langmuir, showing the variation of $(C_D\,Re/24)$ for a sphere with the Reynolds number Re. A rough but adequate approximation to these calculated values, over the whole range of values of r, is given by the empirical formula

$$1/V = \left[1.2 \times 10^6\, r^2\right]^{-1} + \left[2.4 \times 10^3 \sqrt{r}\,\right]^{-1}, \quad (4.17)$$

which tends to the correct limiting forms both when r is very small and very large. The terminal velocity of a large water droplet falling through air is in fact rather lower than the corresponding value for a rigid sphere, since the droplet is deformed by the aerodynamic forces, and better agreement with experimental observations on large rain drops is obtained if the coefficient 6 in equation (4.15) is reduced to about 2.5. Further, it is rather more convenient as regards the subsequent calculations to derive a formula analogous to (4.17) in terms of the relative velocity v; since the difference between the velocities V and v is only important when r is small, the appropriate empirical relation is clearly

$$1/v = \left[1.2 \times 10^6\,(r^2 - a^2)\right]^{-1} + \left[1.6 \times 10^3 \sqrt{r}\,\right]^{-1}. \quad (4.18)$$

(d) <u>The growth of the droplets.</u> Assuming a mean constant value $\bar{E}$ of the efficiency of collection E, the distance fallen by the larger droplet is, by equation (4.11),

$$s \sim 4(r - r_0)/W\bar{E}. \quad (4.19)$$

Also, from equation (4.10), the time of fall to radius r is given by

$$0.3 \times 10^6\, W\bar{E}t = \frac{1}{2a}\log_e\left\{\frac{(r-a)(r_0+a)}{(r+a)(r_0-a)}\right\} + 1500\left[\sqrt{r} - \sqrt{r_0}\,\right]. \quad (4.20)$$

When r is large compared with a, this equation reduces to

$$0.3 \times 10^6\, W\bar{E}\,(t - t_1) = 1500\left[\sqrt{r} - \sqrt{r_0}\,\right] - (1/r), \quad (4.21)$$

where

$$0.3 \times 10^6\, W\bar{E}\, t_1 = (1/2a)\log_e\left[(r_0 + a)/(r_0 - a)\right], \quad (4.22)$$

and, expanding the logarithm as a power series, this becomes

$$t_1 = \left[0.3 \times 10^6\, W\bar{E}\, r_0\right]^{-1}\left\{1 + \tfrac{1}{3}\left(\frac{a}{r_0}\right)^2 + \tfrac{1}{5}\left(\frac{a}{r_0}\right)^4 \ldots\right\}. \quad (4.23)$$

The second factor on the right of equation (4.23) shows the effect on t_1, and hence on the time of fall, of the initial relative size of the larger and smaller droplets, an effect which only becomes important when (a/r_0) is only slightly less than unity. For example, when $a/r_0 = 0.25$, this factor is only about 1.02, when $a/r_0 = 0.5$ it is about 1.1, and when $a/r_0 = 0.75$ it is still only about 1.3. Therefore, since in fact the initial size of the droplets is distributed continuously over a range of values, it would appear reasonable to adopt an average value

$$t_1 \sim k\left[0.3 \times 10^6\, W\bar{E}\, r_0\right]^{-1}, \quad (4.24)$$

where k is an empirical constant which is probably of order 1.1. - 1.2. Assuming provisionally that $W \sim 3.5 \times 10^{-4}$, $\overline{E} \sim 0.2$ and $r_0 \sim 10$ microns, t_1 is of order 60 secs. It should be noted also that t_1 is inversely proportional to the initial radius r_0 of the larger droplet, and would thus, for example, be of order 10 minutes for a droplet of 1 micron radius falling through a mist of still smaller droplets, assuming of course the same values of W and $\overline{E}$. Coagulation due to differential falling velocities is therefore only of importance after a significant proportion of relatively large droplets have been formed by other mechanisms of droplet growth.

Table 1 below shows the time and distance of fall, as functions of the radius of the larger droplet, for the assumed values of the parameters, W, $\overline{E}$ and r_0 stated above.

Table 1.

Radius r (microns)	Time t (seconds)	Distance s (metres)
10	0	0
25	42	0.9
50	53	2.3
100	60	5.1
250	67	14
500	73	28
1000	80	57
2500	93	140

This table shows clearly that the greater part of the time of fall elapses whilst the droplet is still relatively small, less than 100 microns radius, and that almost the whole of its growth occurs during the last 20 seconds of fall. Similarly, almost the whole distance of fall is covered during the last 20 seconds of fall. The table has been terminated at a droplet radius r of 2500 microns, since a larger raindrop is unstable and, if formed, would break up into a number of smaller drops.

Let us now consider in rather more detail the values which should be assigned to the parameters W, $\overline{E}$ and r_0 in the equations determining the time and distance of fall. We shall assume that the rate of coagulation due to differential falling velocities is relatively negligible until the average droplet has attained a radius of 4 microns, that is, at a time of about 1 minute after the formation of the base surge, and that the distribution of droplet sizes is then given by the formulae developed in Section 2. Assuming that the total concentration of liquid water in the mist is 3.5×10^{-4} gm./cm. , the concentration of droplets corresponding to a mean radius of 4 microns is $N \sim 1.3 \times 10^6$ per c.c. The initial concentration of droplets of radius $r \geq 8$ microns is therefore $N e^{-8} \sim 4 \times 10^2$ per c.c., which contain $9 e^{-8} \sim 0.3\%$ of the total liquid water content of the mist. The corresponding values for droplets of radius $r \geq 10$ microns are 2×10^{-1} per c.c. and $3 \times 10^{-4}\%$ of the total liquid water content; for $r \geq 12$ microns, the values are 10^{-6} per c.c. and 5×10^{-9} %. These values show that the relatively few droplets whose initial radius is greater than 12 microns could have made only an insignificant contribution to the total observed rainfall from the Bikini base surge, for, even if each of these droplets had grown to a radius of 2500 microns, they would then have contained only about 0.02% of the total liquid water content of the base surge. The appropriate initial value r_0 of the radius of the larger droplets is therefore almost certainly less than 12 microns.

The cross-sectional area of an 8 micron radius droplet is about 2×10^{-6} cm.^{2}, and hence the total horizontal projection of the 8 micron or larger droplets contained in 1 c.c. of the mist is about 8×10^{-4}cm.^{2}. Therefore, including an additional factor $(1/e)$ to allow for the fact that these areas overlap to some extent, the horizontal area of the base surge is covered by the droplets of radius greater than 8 microns contained within a column about 30 metres high. This distance is small compared with the height of the base surge, which is of order 400 metres, and hence it follows that the surge contains sufficient droplets of radius greater than 8 microns to capture almost all smaller droplets, and consequently cause almost complete precipitation of the water content as rain. The corresponding equivalent height for the droplets of radius greater than 10 microns is of order 5×10^4 metres and, even allowing for the growth of such droplets during their fall, it would appear doubtful that there are sufficient of them to cause total precipitation from the base surge. The appropriate value of r_0 would therefore appear to lie within the close limits 8 - 10 microns, so the assumption r_0 = 10 microns, which was adopted for the calculations shown in Table 1, would appear to be both reasonable and adequate.

During the early stages of coagulation due to differential falling velocities, the droplets of radius greater than 8 microns are falling through a mist of smaller droplets which contain almost the whole of the liquid water content of the surge. Even if each of these larger droplets attains a radius of 50 microns, and this is certainly a gross overestimate since some would be captured during the process of growth, they would still only contain about 60% of the total liquid water. According to Table 1, a radius of 50 microns is attained after a time of fall of 53 secs.; the assumption that $W \sim 3.5 \times 10^{-4}$, that is, that the smaller droplets contain the total liquid water content of the surge, should therefore be an adequate approximation during at least the first half of the total time of fall. On the other hand, the appropriate value of W during the later stages of growth should apparently be smaller, since the larger droplets would then be falling through a region which had already been partially swept by preceding large droplets, although this effect will be to some extent offset by mutual capture amongst the larger droplets themselves. Hazarding a guess, the duration of the later stages of growth may have been thereby perhaps about doubled.

Against this, the average efficiency of collection during the later stages of growth, that is for r in the range 100 - 2500 microns, is probably about double the assumed value 0.2; this follows from the theoretical calculations of Langmuir, whose results are probably fairly accurate for such large droplets. Therefore, since the time of fall is inversely proportional to $\overline{E}$, the duration of the later stages of growth is reduced once more to about half a minute.

The crucial unchecked assumption on which the numerical estimates have been based is of course the value of the efficiency of collection during the early stages of growth, for the total time of fall is dominated by the time to a radius of 100 microns, or even to a radius of only 25 microns. It is unlikely that the assumed value $\overline{E}$ = 0.2 is too low, but it should be appreciated that it may be a gross overestimate; if this is in fact the case, the actual times of fall would be very much greater than the tabulated values. The only possible conclusion in such a case would be that the initial mist contained enough droplets of radius 10 - 20 microns to act as seeds for precipitating the rain. We believe that this possibility deserves serious consideration, and the following Section 5 considers the point further.

(c) Precipitation of the water content of the base surge as rain. On the basis of a number of assumptions, some of which are of unknown validity, it has been estimated above that an appreciable part of the liquid water content of the Bikini base surge should have coagulated to large raindrops after a total time of about $2\frac{1}{2}$ minutes from the time of formation of the column of mist. It is therefore estimated that heavy rain should have commenced to fall from the bottom of the surge at about $2\frac{1}{2}$ minutes after the explosion. The large raindrops which had then been formed nearest to the top of the surge would have fallen at a constant terminal velocity of about 8 metres/sec., and should therefore have reached the ground after about a further $\frac{1}{2}$ - 1 minute. The heavy rain should therefore have been of quite short duration, probably only about $\frac{1}{2}$ - 1 minute, and at the end of this time most of the liquid water content of the surge should have been precipitated.

Experimentally, it was observed at Bikini that heavy rain began to fall from the base surge at about 3 minutes after the explosion. The surge then rapidly slowed down, and finally stopped spreading altogether after a further 2 minutes. We infer that the density of the surge was by then not significantly greater than the density of the surrounding air. The agreement between the theoretical estimates and the experimental observations is thus remarkably good, although probably at least partly fortuitous.

5. An Inversion of the Previous Arguments

Earlier in this paper, we have attempted to estimate the time at which rain begins to fall out of a mist which initially consisted only of very fine drops of water, the initial radius being of the order of 10^{-5} cm. In the development of the theory we encountered a major uncertainty, namely the probability that a larger droplet falling and overtaking a smaller droplet would actually collide with and capture the smaller droplet. Equation (4.12), which admittedly is not exact, suggests that coalescence will not occur at all unless the square of the radius of the smaller droplet multiplied by the radius of the larger droplet is at least 6×10^{-10} cm.^{3}. The equation however does give some basis to the possibility that particles of radii 5 - 10 microns do not readily collide and combine. Of course, the equation fails completely for much smaller particles, since from work on the coagulation of fine particles (of radius 0.1 microns or even less) it is known that experimental results require that the particles collide and coalesce in a manner directly calculable from the Brownian motion treatment of the Kinetic Theory of Gases. Our difficulty is that we can say with some degree of confidence that Brownian motion and differential evaporation and condensation processes must in any case bring the average particle radius up to roughly 5 microns at one or two minutes, however small the initial radius. These considerations lead to the conclusion that not enough larger particles are formed to capture the 5 micron particles, thus leading to rain, unless formula (4.12) is not exact. In particular, the collection efficiency for a 10 micron particle overtaking a 5 micron particle should be of the order 10 - 20%. If this should prove to be untrue, then we are forced to the conclusion that the base surge mist must have contained at the start some droplets of radius 20 microns or more.

We are led to consider whether our earlier arguments can be inverted in such a way that some information about the initial particle sizes can be deduced, accepting the hypothesis that the collection efficiency is zero unless the product condition on the radii, already mentioned, is satisfied.

Let us see what follows if we assume that the initial particles were all of radius 20 - 50 microns. Then from the theory given in Section 4d we find that after 1 minute only large raindrops are present. At one minute after detonation, the surge has reached a radius of about $\frac{1}{2}$ mile. The height of the spreading surge is of the order 1000 feet; the rate of fall of large raindrops is about 25 ft./sec. Hence after another half minute, all, or very nearly all, of the water should have fallen out. This was not what actually happened.

Similarly, if one assumes that the initial particle size distribution was such that most particles were of 5 microns radius but a few present were of 20 - 50 microns radius, the fall out should still be complete $1\frac{1}{2}$ minutes after detonation. This hypothesis is therefore equally unacceptable.

An assumption which will meet the facts is that the bulk of the water particles in the mist initially were small, say 1 micron or less, but a small fraction of the total amount of water was in the form of larger drops of radius 20 - 500 microns. These larger droplets were unable to capture any of the smaller ones until Brownian movement and evaporation processes had raised the average radius of the smaller particles to roughly 5 microns, and the time taken to reach this stage was 1 - 2 minutes. The larger drops could then capture smaller ones and at the end of another minute were falling as large raindrops. They reached the surface of the sea in another half minute. The percentage of the total water content of the surge in the form of drops 20 - 500 microns radius could not have been more than 10%, and might have been considerably less. The base surge precipitates about 1 cm. depth of rain over the whole area covered (7 square miles) The least amount of water initially present in the surge in the form of droplets 25 microns radius that will "seed out" all the water as rain is about 1% by volume.

To summarise our conclusions we have:-

1. The initial radius of the water droplets in the "water column" thrown into the air is unknown, but the bulk of the water was in the form of very fine droplets 1 micron or less in diameter.

2. One minute after detonation, at which time the base surge radius is only half a mile, the bulk of the water is in the form of droplets 1 - 5 microns in radius. Between 1 and 10 per cent of the water must be in the form of large drops at least 25 microns radius, if the capture efficiency product rule is true; otherwise, little or none of the water at this stage is in the form of droplets as large as 25 microns.

3. Two minutes after detonation, there are enough 5 - 10 micron particles resulting from the coagulation of the original fine particles to permit the growth of large drops, either by overtaking and collision among themselves if the capture efficiency product rule is not true; or through capture by the 25 micron and larger particles present from the start, if the capture efficiency product rule is true.

4. Three minutes after detonation, and for the following two minutes, precipitation of rain on the sea takes place, and the base surge stops spreading. Some of the larger of the fine particles present from the start have escaped collision, but not evaporation, processes. They are too small to be effectively cleared through capture by the rain, and therefore survive. They remain as a fog over the area covered by the surge, and their subsequent history depends on the prevailing meteorological conditions. We are unable to say how much of the total water content of the surge remains in this fog, but it would appear to be less than one per cent.

SUPPLEMENT

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The Base Surge: The Mechanism of Fall-out

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The efficiency of collection of small droplets by a larger drop has been discussed more recently by P.K.Das ("The growth of cloud droplets by coalescence". Indian Journal of Meteorology and Geophysics, Vol.1, No.2, April 1950, pp. 137 - 144). In his original work, Langmuir took account of the inertial and drag forces acting on the smaller droplets in the field of flow around the larger drop, whereas Das has allowed in addition for the finite size of the smaller droplets. It is clear that this additional factor will increase the efficiency of collection, since some smaller droplets whose centres would not touch the larger drop will be captured. In fact, since the efficiency of collection E is defined in terms of the number of smaller droplets within the volume swept out by the larger drop, E can now be greater than 100%. A further factor, namely the effect of the smaller droplet on the flow pattern around the larger drop, is not taken into account by either Langmuir or Das; it can reasonably be argued that this effect will increase E still further, but a quantitative treatment would be very difficult.

Das has calculated numerical values of E over a range of values of (a/r) and k, where a and r are respectively the radii of the smaller and larger droplets, and the non-dimensional parameter k is roughly proportional to (a^2r) when both drops are small. Comparison with Langmuir's earlier results shows that the efficiency of collection is increased very considerably in the new approach. The increase is most marked at smaller values of k, showing that with very small droplets almost all capture occurs as a result of glancing blows. It is of course assumed by both Langmuir and Das that capture occurs whenever two droplets come into contact with each other.

As an illustration of the new theory, we shall consider the capture of 3 micron radius droplets by a 10 micron radius drop. Then $a/r = 0.3$ and $k \sim 0.1$, and hence, according to Das,

$$E \text{ (Langmuir)} = 1\% \quad \text{and} \quad E \text{ (Das)} = 89\%$$

It must however be noted that these results are based on the assumption of potential flow past the larger drop, whereas the actual flow past a 10 micron drop is more nearly of Stokesian type. Both Langmuir and Das agree that this would reduce the efficiency of collection, and in fact Langmuir has then calculated that $E = 0$ if k is less than about unity. The corresponding results on Das' theory have not yet been worked out, but the whole trend of his present results suggest that, for $k \sim 0.1$, E can hardly be less than about 0.1.

In our calculations on the mechanism of fall-out from the base surge, the main difficulty was that Langmuir's theory predicted zero efficiency of collection during the crucial early stages of growth by differential falling velocities; the new results obtained by Das would appear to eliminate this difficulty entirely. We assumed a constant average value E = 0.2, which may be incorrect by a factor of 2 or 3 but can hardly be wrong by a factor of 10; such an error is well within the over-all accuracy of the whole calculation.